$\therefore$ An anharmonic ratio $\dfrac{AM}{MN}:\dfrac{AB}{BN}=\dfrac{c^2-y_1^2}{x_1^2-c^2}$; but $c^2=\dfrac{a^2b^2}{a^2+b^2}$.

$\therefore$ An anharmonic ratio$=\dfrac{b^2}{a^2}\cdot\dfrac{(a^2+b^2)x_1^2-a^2b^2}{(a^2+b^2)x_1^2-a^2b^2}=\dfrac{b^2}{a^2}$.

Also solved by G. B. M. Zerr.

CALCULUS.

181. Proposed by S. F. NORRIS, Baltimore, Md.

Integrate $dy=\dfrac{x^2dx}{1+x^4}$. [From Olney's *Integral Calculus*, page 116, third example, second part].

Solution by G. W. GREENWOOD, M. A.; M. E. GRABER, M. A., and G. B. M. ZERR, A. M., Ph. D.

$$\frac{x^2}{1+x^4}=\frac{1}{2\sqrt{2}}\left[\frac{x}{x^2-x\sqrt{2}+1}-\frac{x}{x^2+x\sqrt{2}+1}\right]$$

$$=\frac{1}{4\sqrt{2}}\left[\frac{(2x-\sqrt{2})+\sqrt{2}}{x^2-x\sqrt{2}+1}-\frac{(2x+\sqrt{2})-\sqrt{2}}{x^2+x\sqrt{2}+1}\right]$$

$$=\frac{1}{4\sqrt{2}}\left[\frac{2x-\sqrt{2}}{x^2-x\sqrt{2}+1}-\frac{2x+\sqrt{2}}{x^2+x\sqrt{2}+1}\right]$$

$$+\tfrac{1}{4}\left[\frac{1}{x^2-x\sqrt{2}+1}+\frac{1}{x^2+x\sqrt{2}+1}\right].$$

The required integral is therefore

$$=\frac{1}{4\sqrt{2}}[\log(x^2-x\sqrt{2}+1)-\log(x^2+x\sqrt{2}+1)]$$

$$+\frac{\sqrt{2}}{4}[\tan^{-1}(\sqrt{2}x-1)+\tan^{-1}(\sqrt{2}x+1]$$

$$=\frac{1}{4\sqrt{2}}\log\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}+\frac{\sqrt{2}}{4}\tan^{-1}\frac{x\sqrt{2}}{1-x^2}.$$

Also solved by J. Scheffer.

182. Proposed by A. H. HOLMES, Brunswick, Maine.

Evaluate $\displaystyle\int_0^{\frac{1}{2}\pi}d\theta\sqrt{[1+\sin^2\theta(1-4\cos\theta)]}$.